

Lunar Meteorite Impact Risks



Based on research by Dr. Bill Cooke, a member of NASA's Space Environments team, one danger to lunar explorers and their equipment will be meteorite impacts (reported on CNN.com December 4, 2006). Between November 2005 and November 2006, Dr. Cooke's team looked for and found lunar flashes resulting from meteoroid impacts (red dots on the image); the team found 12 of these events in a single year. The flashes were caused primarily by large Leonid meteors about 7.6 cm across, impacting the Moon with the equivalent energy of 150-300 pounds of TNT. For more information, see: http://science.nasa.gov/headlines/y2006/01dec_lunarleonid.htm

National Science Standards Grades 9-12

Content Standard A:

1. Students will develop abilities necessary to do scientific inquiry, including the ability to design and conduct a scientific investigation and the ability to use mathematics to improve the investigation.
2. To understand scientific inquiry, students will learn about technology which enhances the gathering and manipulation of data (e.g., the technology that made possible the observation of the lunar flashes)

Content Standard B:

All students should develop an understanding of the interactions of energy and matter. In this exercise, students are told about impacts from 7.6 cm meteoroids that release an equivalent energy of 150 to 300 pounds of TNT. Students will calculate that energy, assuming direct conversion of kinetic energy.

Program Standard C:

The science program should be coordinated with the mathematics program to enhance student use and understanding of mathematics in the study of science and to improve student understanding of mathematics.

This exercise, *Lunar Meteorite Impact Risks* specifically addresses the following examples of mathematics that students should be able to use and understand. (See <http://www.nap.edu/readingroom/books/nses/7.html#spc> for more information.)

1. Students will understand connections within a problem and its model as a function in symbolic form (e.g., surface area of a sphere).
2. Students will use functions that are constructed as models of real-world problems.
3. Students will know how to use statistics and probability.

Questions

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Diameter of the Moon: 3,476 kilometers

Problem 1: From the formula for the surface of a sphere ($4\pi r^2$), what is the area, in square kilometers, of the side of the moon facing Earth?

Problem 2: Although an actual impact only affects the few square meters within its immediate vicinity, we can define an impact area as the total area of the surface being struck, by the number of objects striking it. What was the average impact area for a single event?

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts (assume Dr. Cooke observed 100% of the time)?

Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Problem 6: From the image, the impacts do not seem random. If we assume the impacts are clustered into three groups with each group covering an area 700 kilometers on a side with four strikes per group, what is the average impact area?

Problem 7: If you were a colony located in one of these three zones, what would be your answer to Problem 5?

Problem 8: (a) Assuming the speed of an impacting meteoroid is 70 km/s (typical of Leonids), with mass 0.2 g, calculate its kinetic energy in joules and convert to equivalent tons of TNT (1 joule = 2.4×10^{-10} tons TNT). (b) If Dr. Cooke sees impacts with equivalent energy of 200 pounds of TNT, how much more mass must those impactors have?

Answer Key

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Problem 1: From the formula for the surface of a sphere, what is the area, in square kilometers, of the side of the moon facing Earth?

Answer: Surface Area = $4\pi r^2$ We see only about $\frac{1}{2}$ of the Moon, $\frac{1}{2} \times 4\pi \times (1738 \text{ km})^2 = 1.89 \times 10^7 \text{ km}^2$

Problem 2: Although an actual impact only affects the few square meters within its immediate vicinity, we can define an impact zone area as the total area of the surface being struck, by the number of objects striking it. What was the average impact zone area for a single event?

Answer: $1.89 \times 10^7 \text{ km}^2 / 12 = 1.58 \times 10^6 \text{ km}^2$

Problem 3: Assuming the area is a square with a side length 'S', A) what is the length of the side of the impact area? B) What is the average distance between the centers of each impact area?

Answer: A) $S = (1.58 \times 10^6 \text{ km}^2)^{1/2}$ about 1,257 km B) 1,257 km.

Problem 4: If the impacts happen randomly and uniformly in time, about what would be the time interval between impacts (assume Dr. Cooke observed 100% of the time)?

Answer: 1 year / 12 impacts = One month.

Problem 5: From the vantage point of an astronaut standing on the Moon, the horizon is about 3 kilometers away. How long would the lunar colony have to wait before it was likely to see an impact within its horizon area?

Answer: In one year, the impact area is $1.58 \times 10^6 \text{ km}^2$, but the area of the horizon region around the colony is about $\pi \times (3 \text{ km})^2 = 28 \text{ km}^2$. In this area you expect the following number of impacts per year: $1 \text{ impact} / 1.58 \times 10^6 \text{ km}^2/\text{year} \times (28 \text{ km}^2) = 0.000018 \text{ impact per year}$. For this problem however, we want to know the number of years per impact or the reciprocal of the above result = 56,000 years.

Problems 6 and 7: From the image, the impacts do not seem random. If we assume the impacts are clustered into three groups with each group covering an area 700 kilometers on a side with four strikes per group, what is the average impact area?

Answer: Problem 6: $(700 \text{ km}) \times (700 \text{ km}) / 4 = 122,500 \text{ km}^2$

Problem 7: As in problem 5, $1 \text{ impact} / 122,500 \text{ km}^2 \times 28 \text{ km}^2 = 0.00022 \text{ impacts/year}$. The reciprocal then is 4500 years per impact.

Note to Teacher: This calculation assumes that the clustering of impacts is a real effect that persists over a long time. In fact, this is extremely unlikely, and it is statistically probable that when thousands of impacts are plotted, a uniform strike distribution will result. This is similar to the result of flipping a coin 12 times and getting a different outcome than half-Heads and half-Tails.

Problem 8: (a) Convert the mass of the meteoroid to kg: $0.2 \text{ g} = 2.0 \times 10^{-4} \text{ kg}$. Convert the speed of the meteoroid to m/s: $7.0 \times 10^1 \text{ km/s} = 7.0 \times 10^4 \text{ m/s}$. Now kinetic energy = $\frac{1}{2} mv^2$ or, $E = \frac{1}{2} \times 2.0 \times 10^{-4} \text{ kg} \times (7.0 \times 10^4 \text{ m/s})^2 = 4.9 \times 10^5 \text{ joules}$. Now convert $4.9 \times 10^5 \text{ joules}$ to tons of TNT: $4.9 \times 10^5 \text{ joules} \times 2.4 \times 10^{-10} \text{ tons TNT/joule} = 1.2 \times 10^{-4} \text{ tons TNT}$. (b) 200 pounds TNT = 0.1 ton TNT, divide 0.1 ton TNT by $1.2 \times 10^{-4} \text{ tons TNT} = 830$. Therefore, the mass of Dr. Cooke's meteoroid is $830 \times 0.2 \text{ g} = 170 \text{ g}$.